The Mathematical Theory of Communication
International Morse Code

1. The length of a dot is one unit.
2. A dash is three units.
3. The space between parts of the same letter is one unit.
4. The space between letters is three units.
5. The space between words is seven units.
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Harry Nyquist

- Certain Factors Affecting Telegraph Speed. Bell Labs Technical Journal. 1924

\[ W = K \log m \]

Where \( W \) is the speed of transmission of intelligence, 
\( m \) is the number of current values, 
and, \( K \) is a constant.
Ralph Hartley

• *Transmission of Information.* Bell Labs Technical Journal. 1928

\[ H = n \log s \]
\[ = \log s^n. \]
Claude Shannon

- A Symbolic Analysis of Relay and Switching Circuits. Master’s Thesis. MIT. 1937
Claude Shannon
In Appendix 2, the following result is established:

**Theorem 2:**

The only $H$ satisfying the three above assumptions is of the form:

$$H = \sum_{i} p_i \log p_i$$

where $K$ is a positive constant.

This theorem, and the assumptions required for its proof, are in no way necessary for the present theory. It is given chiefly to lend a certain plausibility to some of our later definitions. The real justification of these definitions, however, will reside in their implications.

Quantities of the form $H \sum p_i \log p_i$ (the constant $K$ merely amounts to a choice of a unit of measure) play a central role in information theory as measures of information, choice and uncertainty. The form of $H$ will be recognized as that of entropy as defined in certain formulations of statistical mechanics where $p_i$ is the probability of a system being in cell $i$ of its phase space. $H$ is then, for example, the $H$ in Boltzmann's famous $H$ theorem. We shall call $H \sum p_i \log p_i$ the entropy of the set of probabilities $p_1 \ldots p_n$. If $x$ is a chance variable we will write $H_x$ for its entropy; thus $x$ is not an argument of a function but a label for a number, to differentiate it from $H_y$ say, the entropy of the chance variable $y$.

The entropy in the case of two possibilities with probabilities $p$ and $q$, namely $H_p \log p + H_q \log q$ is plotted in Fig. 7 as a function of $p$.

The quantity $H$ has a number of interesting properties which further substantiate it as a reasonable measure of choice or information.

1. $H = 0$ if and only if all the $p_i$ but one are zero, this one having the value unity. Thus only when we are certain of the outcome does $H$ vanish. Otherwise $H$ is positive.

2. For a given $n$, $H$ is a maximum and equal to $\log n$ when all the $p_i$ are equal (i.e., $1/n$). This is also intuitively the most uncertain situation.

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**Claude Shannon**


$$H = - \sum p_i \log p_i$$
The Noisy Channel Model

- **Information Source**
- **Transmitter**
- **Channel**
- **Receiver**
- **Destination**

**Diagram Description**
- **Message** from the Information Source is sent to the Transmitter.
- The Transmitter produces a signal suitable for transmission over the channel.
- The channel transmits the signal from the Transmitter to the Receiver.
- The Receiver reconstructs the message from the signal.
- The message is intended for the Destination.

**Noise Source**

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Ad e c i m a ld i g i ti sa b o u t 3

1 bit. A digit wheel on a desk computing machine has ten stable positions and therefore has a storage capacity of one decimal digit. In analytical work where integration and differentiation are involved the base \( e \) is sometimes useful. The resulting units of information will be called natural units.

Change from the base \( a \) to base \( b \) merely requires multiplication by \( \log_b a \).

By a communication system we will mean a system of the type indicated schematically in Fig. 1. It consists of essentially five parts:

1. An **information source** which produces a message or sequence of messages to be communicated to the receiving terminal. The message may be of various types: (a) A sequence of letters as in a telegraph or teletype system; (b) A single function of time \( f(t) \) as in radio or telephony; (c) A function of time and other variables as in black and white television — here the message may be thought of as a function \( f(x, y, t) \) of two space coordinates and time, the light intensity at point \( x, y \) and time \( t \) on a pickup tube plate; (d) Two or more functions of time, say \( f(t), g(t), h(t) \) — this is the case in "three-dimensional" sound transmission or if the system is intended to service several individual channels in multiplex; (e) Several functions of several variables — in color television the message consists of three functions \( f(x, y, t), g(x, y, t), h(x, y, t) \) defined in a three-dimensional continuum — we may also think of these three functions as components of a vector field defined in the region — similarly, several black and white television sources would produce "messages" consisting of a number of functions of three variables; (f) Various combinations also occur, for example in television with an associated audio channel.

2. A **transmitter** which operates on the message in some way to produce a signal suitable for transmission over the channel. In telephony this operation consists merely of changing sound pressure into a proportional electrical current. In telegraphy we have an encoding operation which produces as sequence of dots, dashes and spaces on the channel corresponding to the message. In a multiplex PCM system the different speech functions must be sampled, compressed, quantized and encoded, and finally interleaved properly to construct the signal. Vocoder systems, television and frequency modulation are other examples of complex operations applied to the message to obtain the signal.

3. The **channel** is merely the medium used to transmit the signal from transmitter to receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc.

4. The **receiver** ordinarily performs the inverse operation of that done by the transmitter, reconstructing the message from the signal.

5. The **destination** is the person (or thing) for whom the message is intended.

We wish to consider certain general problems involving communication systems. To do this it is first necessary to represent the various elements involved as mathematical entities, suitably idealized from their
The Noisy Channel Model

Fig. 1—Schematic diagram of a general communication system.

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The Noisy Channel Model

INFORMATION SOURCE

TRANSMITTER

MESSAGE

SIGNAL

RECEIVED SIGNAL

RECEIVER

MESSAGE

DESTINATION

Message encoded in Morse code
The Noisy Channel Model

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**Encoded message corrupted by noise from the transmission lines**
The Noisy Channel Model

The diagram represents a general communication system as follows:

1. **Information Source**: Produces a message or sequence of messages to be communicated to the receiving terminal.
2. **Transmitter**: Operates on the message in some way to produce a signal suitable for transmission over the channel.
3. **Channel**: Transmits the signal from the transmitter to the receiver.
4. **Receiver**: Receives the signal and performs the inverse operation of the transmitter, reconstructing the message.
5. **Destination**: The person (or thing) for whom the message is intended.

The diagram includes a noise source that affects the signal, and a received message is decoded from Morse code.
The Noisy Channel Model

The schematic diagram of a general communication system consists of essentially five parts:

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Received message written in natural language. May contain errors.
The Noisy Channel Model
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The Noisy Channel Model

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The Noisy Channel Model

![Diagram of a general communication system with labeled parts: Information Source, Transmitter, Channel, Receiver, Destination.]

In the diagram:
- **Information Source**: Produces the message or sequence of messages.
- **Transmitter**: Operates on the message to produce a signal suitable for transmission.
- **Channel**: Transmits the signal from the transmitter to the receiver.
- **Receiver**: Reconstructs the message from the received signal.
- **Destination**: Receives the message.

The diagram illustrates the flow of information through the communication system, including the addition of noise by a noise source.
The Noisy Channel Model

Figure 1—Schematic diagram of a general communication system.

- **Information Source** (e) produces a message or sequence of messages to be communicated to the receiving terminal.
- **Transmitter** operates on the message in some way to produce a signal suitable for transmission over the channel.
- **Channel** is merely the medium used to transmit the signal from transmitter to receiver.
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**Symbol:** e, f
The Noisy Channel Model

\[ \hat{e} = \arg \max_{e} p(e|f) \]

INFORMATION SOURCE \[ e \]

TRANSMITTER

SIGNAL

RECEIVER

MESSAGE

RECEIVED SIGNAL

NOISE SOURCE

DESTINATION \[ f \]

MESSAGE
The Noisy Channel Model

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Fig. 1—Schematic diagram of a general communication system.

- **Information Source**
- **Message**
- **Transmitter**
- **Signal**
- **Receiver**
- **Signal Received**
- **Noise Source**
- **Message**
- **Destination**

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The Noisy Channel Model

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**INFORMATION SOURCE**

**TRANSMITTER**

**CHANNEL**

**RECEIVER**

**DESTINATION**

**MESSAGE**

**SIGNAL**

**RECEIVED SIGNAL**

**MESSAGE**

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The Noisy Channel Model

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Diagram:
- Information Source
- Transmitter
- Receiver
- Destination
- Message
- Signal
- Received Signal
- Noise Source
The Noisy Channel Model

\[ \hat{e} = \arg \max_{e} p(e|f) \]

- **Information Source**
- **Transmitter**
- **Channel**
- **Receiver**
- **Destination**

The message is encoded and transmitted, and the noisy channel is depicted with noise added to the signal.

Mathematically, the model represents the decision process at the receiver, where \( \hat{e} \) is the estimated message given the received signal \( f \). This formulation is foundational in information theory, particularly in the study of how messages can be transmitted reliably in the presence of noise.
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\[ \hat{e} = \operatorname{arg \ max} \ p(e|f) \]

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The Noisy Channel Model

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1. **Information Source** which produces a message or sequence of messages to be communicated to the receiving terminal. The message may be of various types:
   - A sequence of letters as in a telegraph or teletype system;
   - A single function of time \( f(t) \) as in radio or telephony;
   - A function of time and other variables as in black and white television — here the message may be thought of as a function \( f(x, y, t) \) of two space coordinates and time, the light intensity at point \( x, y \) and time \( t \) on a pickup tube plate;
   - Two or more functions of time, say \( f(t), g(t), h(t) \) — this is the case in "three-dimensional" sound transmission or if the system is intended to service several individual channels in multiplex;
   - Several functions of several variables — in color television the message consists of three functions \( f(x, y, t), g(x, y, t), h(x, y, t) \) defined in a three-dimensional continuum — we may also think of these three functions as components of a vector field defined in the region — similarly, several black and white television sources would produce "messages" consisting of a number of functions of three variables; and
   - Various combinations also occur, for example in television with an associated audio channel.

2. **Transmitter** which operates on the message in some way to produce a signal suitable for transmission over the channel. In telephony this operation consists merely of changing sound pressure into a proportional electrical current. In telegraphy we have an encoding operation which produces a sequence of dots, dashes and spaces on the channel corresponding to the message. In a multiplex PCM system the different speech functions must be sampled, compressed, quantized and encoded, and finally interleaved properly to construct the signal. Vocoder systems, television and frequency modulation are other examples of complex operations applied to the message to obtain the signal.

3. **Channel** is merely the medium used to transmit the signal from transmitter to receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc.

4. **Receiver** ordinarily performs the inverse operation of that done by the transmitter, reconstructing the message from the signal.

5. **Destination** is the person (or thing) for whom the message is intended.

We wish to consider certain general problems involving communication systems. To do this it is first necessary to represent the various elements involved as mathematical entities, suitably idealized from their actual physical properties.
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- **Information Source**
- **Transmitter**
- **Channel**
- **Receiver**
- **Destination**

**Diagram Details:**
- The diagram represents a general communication system.
- **Message** flows from the **Information Source** through the **Transmitter** to the **Channel**.
- **Received Signal** is then processed by the **Receiver** to produce the **Message**.
- The **Error-Free Transmitter** is indicated by the red arrow.
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### Diagram

- **Information Source**: \( e \)
- **Transmitter**: 
- **Channel**: 
- **Receiver**: 
- **Destination**: \( f \)
- **Signal**: 
- **Received Signal**: 
- **Message**: 
- **Error-free Transmitter**: 
- **Error-free Receiver**: 
- **Noise Source**: 

**Fig. 1**—Schematic diagram of a general communication system.
The Noisy Channel Model

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**Fig. 1—Schematic diagram of a general communication system.**

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p(e|f) = \frac{p(f|e)p(e)}{p(f)}
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\hat{e} = \arg \max_e p(e|f)
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INFORMATION SOURCE \[ \rightarrow \] TRANSMITTER \[ \rightarrow \] RECEIVER \[ \rightarrow \] DESTINATION

MESSAGE \[ \rightarrow \] SIGNAL \[ \rightarrow \] RECEIVED SIGNAL \[ \rightarrow \] MESSAGE

\[ p(e) \rightarrow \] \[ p(f|e) \rightarrow \] NOISE SOURCE
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Fig. 1—Schematic diagram of a general communication system.

By a communication system we will mean a system of the type indicated schematically in Fig. 1. It consists of essentially five parts:

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