## MT Marathon 2016 - Written Exam With Solutions

Please answer the questions directly on this paper. Note that the answers will be checked by some other participant.

The final exam score is a complex number - some questions from the exam were not discussed on the slides, but are important NN topics. These questions are written in italics and are awarded with imaginary points.

You can get a hint for any question (for example if you are unsure how to compute softmax or how to derivate it) - just raise your hand and I will come to you. You will be awarded only $50 \%$ of points for such questions.

1) Compute BLEU (up to bigrams) and TER of the following translations.

Human Translation: the cat jumped on the table
Machine Translation: the table the the cat
$\mathrm{BLEU}=$ brevity penalty $\cdot \exp ($ mean of clipped log probability of unigrams and bigrams)

$$
=e^{1-\frac{6}{5}} \cdot e^{\frac{1}{2} \log \left(\frac{1}{5}\right)+\frac{1}{2} \log \left(\frac{2}{5}\right)}=\sqrt{\frac{2}{5}} e^{-\frac{1}{5}}
$$

TER $=$ word edit distance normalized by length of HT

$$
=\frac{4}{5}
$$

2) Compute the output of the following neural networks.

3) Compute derivatives of the networks with respect to all weights and inputs.

For the first network, the gold output is 3 , and the loss function is MSE, i.e., (output - gold $)^{2}$.


For the second network, the gold output is the distribution $(0,0,1)$ and the loss function is crossentropy (which in the single correct class case is equal to $\log$ probability of the correct class).


Let $g=\left(g_{1}, g_{2}, g_{3}\right)$ be the gold output distribution.
Then $L=-\sum_{j} g_{j} \log o_{j}$. We start by showing
(the well known fact) that $\frac{\partial L}{\partial z_{i}}=o_{i}-g_{i}$.

$$
\begin{aligned}
\frac{\partial L}{\partial z_{i}} & =\frac{\partial-\sum_{j} g_{j} \log o_{j}}{\left.\partial-\sum_{j} z_{i} z_{j} z_{j}-g_{j} \log \Sigma_{k} e^{z_{k}}\right)}=\frac{\partial-\sum_{j} g_{j} \log \left(e^{z_{j}} / \Sigma_{k} e^{z_{k}}\right)}{\partial z_{i} \sum_{j} g_{j} \log \Sigma_{k} e^{z_{k}}} \\
& =\frac{\partial \sum_{j} g_{j} z_{j}}{\partial z_{i}} \\
& =\sum_{j}\left(g_{j} \frac{1}{z_{k} z_{i}} e^{\Sigma_{j}}\right)-g_{i}=\sum_{j}\left(g_{j} o_{i}\right)-g_{i}=o_{i}-g_{i}
\end{aligned}
$$

The rest is straightforward.

$$
\begin{aligned}
& \frac{\partial L}{\partial w_{1}}=\frac{\partial L}{\partial z_{1}} \frac{\partial z_{1}}{\partial w_{1}}=\frac{\partial L}{\partial z_{1}} x_{1}=\frac{3}{12} \quad \frac{\partial L}{\partial w_{2}}=\frac{\partial L}{\partial z_{1}} \frac{\partial z_{1}}{\partial w_{2}}=\frac{\partial L}{\partial z_{1}} x_{2}=\frac{6}{12} \\
& \frac{\partial L}{\partial w_{3}}=\frac{\partial L}{\partial z_{2}} \frac{\partial z_{2}}{\partial w_{3}}=\frac{\partial L}{\partial z_{2}} x_{1}=\frac{4}{12} \quad \frac{\partial L}{\partial w_{4}}=\frac{\partial L}{\partial z_{2}} \frac{\partial z_{2}}{w_{4}}=\frac{\partial L}{\partial z_{2}} x_{2}=\frac{8}{12} \\
& \frac{\partial L}{\partial w_{5}}=\frac{\partial L}{\partial z_{3}} \frac{\partial z_{3}}{\partial w_{5}}=\frac{\partial L}{\partial z_{3}} x_{1}=\frac{5}{12} \quad \frac{\partial L}{\partial w_{6}}=\frac{\partial L}{\partial z_{3}} \frac{\partial z_{3}}{\partial w_{6}}=\frac{\partial L}{z_{3} x_{2}}=\frac{10}{12} \\
& \frac{\partial L}{\partial x_{1}}=\sum_{j} \frac{\partial L}{\partial z_{j}} \frac{\partial z_{j}}{\partial x_{1}}=\sum_{j} \frac{\partial L}{\partial z_{j}} w_{2 j-1}=\frac{-3 \log 3-4 \log 4+5 \log 5}{12} \\
& \frac{\partial L}{\partial x_{2}}=\sum_{j} \frac{\partial L}{\partial z_{j}} \frac{\partial z_{j}}{\partial x_{2}}=\sum_{j} \frac{\partial L}{\partial z_{j}} w_{2 j}=\frac{3 \log 3+4 \log 4}{12}
\end{aligned}
$$

## 4) Write explicit GRU equations.

Compute $h_{t}$ using $h_{t-1}$ and $x$, using the GRU sketch from the slides. Include all weight matrices and bias terms.


$$
\begin{aligned}
& r_{t}=\sigma\left(W_{r} x+U_{r} h_{t-1}+b_{r}\right) \\
& z_{t}=\sigma\left(W_{z} x+U_{z} h_{t-1}+b_{z}\right) \\
& \tilde{h}_{t}=\sigma\left(W_{h} x+U_{h}\left(r_{t} \odot h_{t-1}\right)+b_{h}\right) \\
& h_{t}=\left(1-z_{t}\right) \odot h_{t-1}+z_{t} \odot \tilde{h}_{t}
\end{aligned}
$$

5) Explain (with formulas) how can we handle the exploding gradient problem.

We can perform gradient clipping, either using the norm of the whole gradient (if $\|g\|_{2}>c$, then $g \leftarrow \frac{c}{\|g\|_{2}} g$ ), or element-wise $\left(\forall i\right.$, if $g_{i}>c$ then $g_{i} \leftarrow c$ ).
6) What is vanishing gradient problem in plain RNNs? Try explaining why it occurs.

The problem is that in plain RNNs the gradient can vanish (become close to zero) during backpropagation.
The problem is caused by the fact that plain RNNs tend to perform nearly the same transformation to the state which is passed through the RNN. If we approximate the derivation of the transformation by its Jacobian matrix J, and if we assume it is the same in all steps, the gradient after the last $i$ steps can be approximated as $J_{i} g$. And if the largest eigenvalue of $J$ is less than one (which may really be the case), the gradient gets smaller and smaller.
7) Assume basic encoder-decoder model without attention. Write/draw the inputs used for decoding the second word during inference time.
Include all operations like tanh, softmax, linear transformations, biases, embeddings, ...

8) Assume encoder-decoder model with attention. Write how exactly is computed the attention $c$ used when decoding the first word.

9) Compute 3 steps and then further 4 steps of BPE on the following dictionary.

| Word | Frequency |
| :--- | :--- |
| t a l l </w> | 2 |
| t a l l e r </w> | 1 |
| t a l l e s t </w> | 1 |
| b e s t </w> | 1 |

The dictionary after 3 steps: tall </w>, tall e r </w>, tall e s t </w>, be s t </w> The dictionary after 7 steps either: tall</w>, tall e r </w>, tall est</w>, b est</w> or: tall</w>, talle r </w>, talle st</w>, b e st</w>
10) Explain the skip-gram model of Mikolov et al.

Consider the basic skip-gram model (which uses full softmax output) with context size 1. Draw the network used in the model, specify exactly how is the output computed (notably it should be clear what all parameters of the model are), and try writing the loss function.


Let $l\left(v_{i}, v_{j}\right)=E_{i, *} O_{*, j}$ is the unormalized output of the network for input word $v_{i}$ and output word $v_{j}$.
The softmax-normalized output is then $s\left(v_{i}, v_{j}\right)=\frac{e^{l\left(v_{i}, v_{j}\right)}}{\sum_{k} e^{\left(l v_{i}, v_{k}\right)}}$. The loss function given $w_{t}$ on input is therefore $L\left(w_{t}\right)=\log \left(s\left(w_{t}, w_{t-1}\right)\right)+\log \left(s\left(w_{t}, w_{t+1}\right)\right)$.

In practice, the full softmax output is too slow. If you know negative sampling, try sketching out how it works.
Instead of normalizing the output words using softmax, let the output for word $v_{i}$ be normalized using the sigmoid function (i.e., the network then computes probability estimate for every word independently). The loss then becomes $\sigma\left(l\left(w_{t}, w_{t-1}\right)\right)-\sum_{\text {random } k \text { words } \mathrm{r}_{\mathrm{i}}} \sigma\left(l\left(w_{t}, r_{i}\right)\right)$. The random negative samples are sampled according to unigram probability ${ }^{3 / 4}$.
Recently, structures skip-gram (SSkip-gram) model is gaining popularity. If we tell you that it uses a separate output matrix for every context offset, try guessing how it is computed.

Instead of sharing the output matrix for every context offset ( -1 and +1 in our case), we have separate output matrices $O_{-1}$ and $O_{1}$. Therefore, the embeddings also encode offset of the context words. The structured skip-gram embeddings can be computed using for example wang2vec.
11) List as many $N N$ training algorithms you know apart from standard $S G D$.

For example SGD with momentum, SGD with Nestorov momentum, AdaGrad, RMSProp, RMSProp with momentum, AdaDelta, Adam; but the list is much larger.

## 12) Name as many $N N$ regularization methods you know.

For example weight decay, dropout, zoneout, BatchNorm, LayerNorm; but again, there are many many others.
13) Why are minibatches regularly used in NN training? Write at least two reasons. One reason is speed: matrix-matrix multiplication is asymptotically faster than performing several individual matrix-verctor multiplications. Also, for GPUs, there is some slowdown for every operation executed (i.e., every network node evaluated), so using batches allow evaluating on multiple inputs. The other reason is accuracy: gradients of individual inputs are quite noisy, so using an averaged gradient of multiple inputs usually results in

