N-Gram Language Modelling including Feed-Forward NNs

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History of Language Model History

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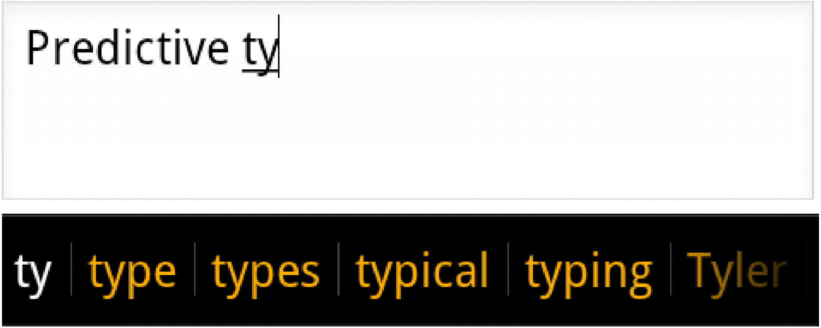
Predictive ty
ty
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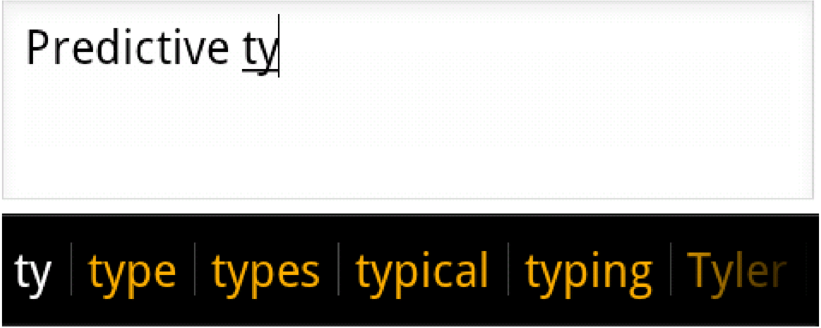
typing
Tyler



$$p(\text{type} \mid \text{Predictive}) > p(\text{Tyler} \mid \text{Predictive})$$

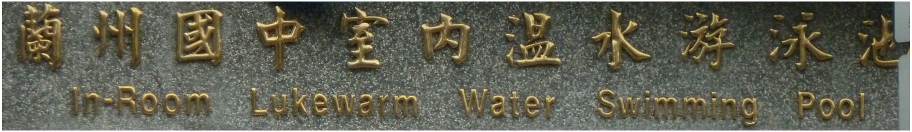
Win or luse, it was a great game.

Win or lose, it were a great game.

Win or loose, it was a great game.

$$p(\text{lose} \mid \text{Win or}) \gg p(\text{loose} \mid \text{Win or})$$

[Church et al, 2007]



Heated indoor swimming pool

Chambre

$\downarrow$

Bedroom

présidente de la Chambre des représentants

$\downarrow$

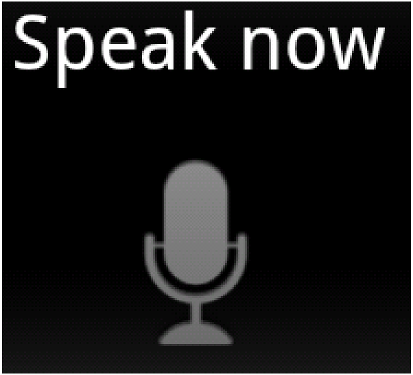
chairwoman of the Bedroom of Representatives présidente de la $\boxed{\text{Chambre des représentants}}$

chairwoman of the $\boxed{\text{House of Representatives}}$

présidente de la $\boxed{\text{Chambre des représentants}}$

chairwoman of the $\boxed{\text{House of Representatives}}$

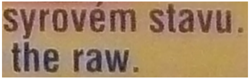
$$p(\text{chairwoman of the House of Representatives})$$
$$>$$
$$p(\text{chairwoman of the Bedroom of Representatives})$$



$$p(\text{Another one bites the dust.})$$
$$>$$
$$p(\text{Another one rides the bus.})$$

Prediction

Spelling



Translation

Speech

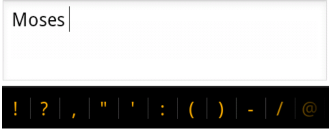
Essential Component: **Language Model**

$$p(\text{Moses Compiles}) = ?$$

Sequence Models

Chain Rule

$$p(\text{Moses compiles}) = p(\text{Moses})\,p(\text{compiles} \mid \text{Moses})$$



Sequence Model

$$
\begin{array}{rll}
 & \log p(\text{iran} & \mid \text{<s>} \quad) \\
 & \log p(\text{is} & \mid \text{<s> iran} \quad) \\
 & \log p(\text{one} & \mid \text{<s> iran is} \quad) \\
 & \log p(\text{of} & \mid \text{<s> iran is one} \quad) \\
 & \log p(\text{the} & \mid \text{<s> iran is one of} \quad) \\
 & \log p(\text{few} & \mid \text{<s> iran is one of the} \quad) \\
 & \log p(\text{countries} & \mid \text{<s> iran is one of the few} \quad) \\
 & \log p(. & \mid \text{<s> iran is one of the few countries} \quad) \\
+ & \log p(\text{</s>} & \mid \text{<s> iran is one of the few countries .}) \\
\hline
= & \multicolumn{2}{l}{\log p(\text{<s> iran is one of the few countries . </s>} \quad)}
\end{array}
$$

Sequence Model

$$
\begin{array}{rllr}
 & \log p(\text{iran} & \mid \text{<s>} &) \\
 & \log p(\text{is} & \mid \text{<s> iran} &) \\
 & \log p(\text{one} & \mid \text{<s> iran is} &) \\
 & \log p(\text{of} & \mid \text{<s> iran is one} &) \\
 & \log p(\text{the} & \mid \text{<s> iran is one of} &) \\
 & \log p(\text{few} & \mid \text{<s> iran is one of the} &) \\
 & \log p(\text{countries} & \mid \text{<s> iran is one of the few} &) \\
 & \log p(\text{.} & \mid \text{<s> iran is one of the few countries} &) \\
+ & \log p(\text{</s>} & \mid \text{<s> iran is one of the few countries .} &) \\
\hline
= & \log p(\text{<s> iran is one of the few countries . </s>} & &)
\end{array}
$$

Explicit begin and end of sentence.

Sequence Model

$$
\begin{array}{rll|lrr}
 & \log p(& \text{iran} & \text{<s>} &)= & \text{-3.33437} \\
 & \log p(& \text{is} & \text{<s> iran} &)= & \text{-1.05931} \\
 & \log p(& \text{one} & \text{<s> iran is} &)= & \text{-1.80743} \\
 & \log p(& \text{of} & \text{<s> iran is one} &)= & \text{-0.03705} \\
 & \log p(& \text{the} & \text{<s> iran is one of} &)= & \text{-0.08317} \\
 & \log p(& \text{few} & \text{<s> iran is one of the} &)= & \text{-1.20788} \\
 & \log p(& \text{countries} & \text{<s> iran is one of the few} &)= & \text{-1.62030} \\
 & \log p(& \text{.} & \text{<s> iran is one of the few countries} &)= & \text{-2.60261} \\
+ & \log p(& \text{</s>} & \text{<s> iran is one of the few countries .} &)= & \text{-0.04688} \\
\hline
= & \log p(& \multicolumn{2}{l}{\text{<s> iran is one of the few countries . </s>}} &)= & \text{-11.79900}
\end{array}
$$

Where do these probabilities come from?

Probabilities from Text

Model

$p(\text{raw} \mid \text{in the})$

Estimating from Text

help in the search for an answer .
Copper burned in the raw wood .
put forward in the paper
Highs in the 50s to lower 60s .

$\vdots$

$$\Longrightarrow \quad p(\text{raw} \mid \text{in the}) \approx \frac{1}{4}$$

Estimating from Text

help in the search for an answer .
Copper burned in the raw wood .
put forward in the paper
Highs in the 50s to lower 60s .

$\vdots$

$$\Longrightarrow$$

$$p(\text{raw} \mid \text{in the}) \approx \frac{1}{4}$$

$$p(\text{Ugrasena} \mid \text{in the}) \approx 0$$

Estimating from Text

help in the search for an answer .
Copper burned in the raw wood .
put forward in the paper
Highs in the 50s to lower 60s .

⋮

$\Longrightarrow$

$$p(\text{raw} \mid \text{in the}) \approx \frac{1}{6}$$
$$p(\text{Ugrasena} \mid \text{in the}) \approx \frac{1}{1000}$$

Problem

"in the Ugrasena" was not seen, but could happen.

$$p(\text{Ugrasena} \mid \text{in the}) = \frac{\text{count(in the Ugrasena)}}{\text{count(in the)}} = 0?$$

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$$= \frac{\text{count(the Ugrasena)}}{\text{count(the)}} = 2.07 \cdot 10^{-9}$$

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$$p(\text{Ugrasena} \mid \text{in the}) = \cancel{\frac{\text{count(in the Ugrasena)}}{\text{count(in the)}}} = 0?$$

$$= \frac{\text{count(the Ugrasena)}}{\text{count(the)}} = 2.07 \cdot 10^{-9}$$

Stupid Backoff: Drop context until count is non-zero
[Brants et al, 2007]

Can we be less stupid?

Smoothing

"in the Ugrasena" was not seen, but could happen.

1. **Backoff**: maybe "the Ugrasena" was seen?
2. **Neural Networks**: classifier predicts next word

Backoff Smoothing

"in the Ugrasena" was not seen $\rightarrow$ try "the Ugrasena"

$$p(\text{Ugrasena} \mid \text{in the}) \approx p(\text{Ugrasena} \mid \text{the})$$

Backoff Smoothing

"in the Ugrasena" was not seen $\rightarrow$ try "the Ugrasena"

$$p(\text{Ugrasena} \mid \text{in the}) \approx p(\text{Ugrasena} \mid \text{the})$$

"the Ugrasena" was not seen $\rightarrow$ try "Ugrasena"

$$p(\text{Ugrasena} \mid \text{the}) \approx p(\text{Ugrasena})$$

Backoff Smoothing

"in the Ugrasena" was not seen $\rightarrow$ try "the Ugrasena"

$$p(\text{Ugrasena} \mid \text{in the}) = p(\text{Ugrasena} \mid \text{the}) b(\text{in the})$$

"the Ugrasena" was not seen $\rightarrow$ try "Ugrasena"

$$p(\text{Ugrasena} \mid \text{the}) = p(\text{Ugrasena}) b(\text{the})$$

Backoff b is a penalty for not matching context.

Backoff Language Model

Unigrams

Words	**log *p***	**log *b***
<s>	$-\infty$	-2.0
iran	-4.1	-0.8
is	-2.5	-1.4
one	-3.3	-0.9
of	-2.5	-1.1

Bigrams

Words	**log *p***	**log *b***
<s> iran	-3.3	-1.2
iran is	-1.7	-0.4
is one	-2.0	-0.9
one of	-1.4	-0.6

Trigrams

Words	**log *p***
<s> iran is	-1.1
iran is one	-2.0
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Trigrams

Words	**log *p***
<s> iran is	-1.1
iran is one	-2.0
is one of	-0.3

Query

$$\log p(\text{is} \mid <\!\text{s}\!> \text{iran}) = -1.1$$

Backoff Language Model

Unigrams

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Trigrams

Words	log p
<s> iran is	-1.1
iran is one	-2.0
is one of	-0.3

$$\text{Query}: \; p(\text{of} \mid \text{iran is})$$

$$\begin{array}{lr}
\log p(\text{of}) & -2.5 \\
\log b(\text{is}) & -1.4 \\
\log b(\text{iran is}) & + \; -0.4 \\
\hline
\log p(\text{of} \mid \text{iran is}) & = \; -4.3
\end{array}$$

Close words matter more.

Though long-distance matters:

- Grammatical structure
- Topical coherence
- Words tend to repeat
- Cross-sentence dependencies

Where do p and b come from? Text!

Kneser-Ney

Witten-Bell

Good-Turing

Kneser-Ney

Common high-quality smoothing

1. Adjust
2. Normalize
3. Interpolate

Adjusted counts are:

Trigrams Count in the text.

Others Number of unique words to the left.

Lower orders are used when a trigram did not match.
How freely does the text associate with new words?

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Others Number of unique words to the left.

Lower orders are used when a trigram did not match.
How freely does the text associate with new words?

Input

Trigam	Count
are one of	1
is one of	5
are two of	3

→

Output

1-gram	Adjusted
of	2

Output

2-gram	Adjusted
one of	2
two of	1

Discounting and Normalization

Save mass for unseen events

$$\text{pseudo}(w_n|w_1^{n-1}) = \frac{\text{adjusted}(w_1^n) - \text{discount}_n(\text{adjusted}(w_1^n))}{\sum_x \text{adjusted}(w_1^{n-1}x)}$$

Normalize

Discounting and Normalization

Save mass for unseen events

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Normalize

Input

3-gram	Adjusted
are one of	1
are one that	2
is one of	5

→

Output

3-gram	Pseudo
are one of	0.26
are one that	0.47
is one of	0.62

Interpolate: Sparsity vs. Specificity

Interpolate unigrams with the uniform distribution.

$$p(\text{of}) = \text{pseudo}(\text{of}) + \text{backoff}(\epsilon)\frac{1}{|\text{vocabulary}|}$$

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Interpolate unigrams with the uniform distribution,

$$p(\text{of}) = \text{pseudo}(\text{of}) + \text{backoff}(\epsilon)\frac{1}{|\text{vocabulary}|}$$

Interpolate bigrams with unigrams, etc.

$$p(\text{of}|\text{one}) = \text{pseudo}(\text{of} \mid \text{one}) + \text{backoff}(\text{one})p(\text{of})$$

Interpolate: Sparsity vs. Specificity

Interpolate unigrams with the uniform distribution,

$$p(\text{of}) = \text{pseudo}(\text{of}) + \text{backoff}(\epsilon)\frac{1}{|\text{vocabulary}|}$$

Interpolate bigrams with unigrams, etc.

$$p(\text{of}|\text{one}) = \text{pseudo}(\text{of} \mid \text{one}) + \text{backoff}(\text{one})p(\text{of})$$

Input

n-gram	pseudo	interpolation weight
of	0.1	backoff(ϵ) = 0.1
one of	0.2	backoff(one) = 0.3
are one of	0.4	backoff(are one) = 0.2

Output

n-gram	p
of	0.110
one of	0.233
are one of	0.447

Kneser-Ney Intuition

Adjust Measure association with new words.
Normalize Leave space for unseen events.
Interpolate Handle sparsity.

How do we implement it?

"LM queries often account for more than 50% of the CPU"
[Green et al, WMT 2014]

500 billion entries in my largest model

Need speed and memory efficiency

Counting *n*-grams

<s> Australia is one of the few

5-gram	**Count**
<s> Australia is one of	1
Australia is one of the	1
is one of the few	1

Hash table?

Counting *n*-grams

<s> Australia is one of the few

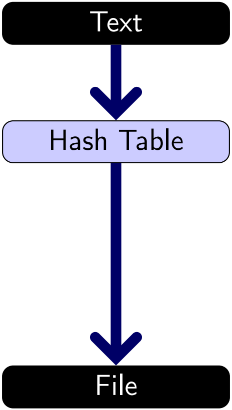
$\Downarrow$

5-gram	**Count**
<s> Australia is one of	1
Australia is one of the	1
is one of the few	1

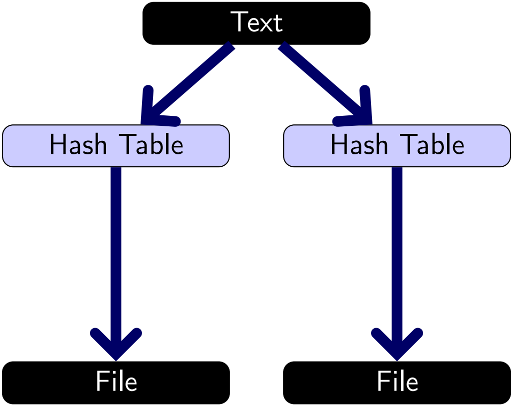
Hash table?

Runs out of RAM.

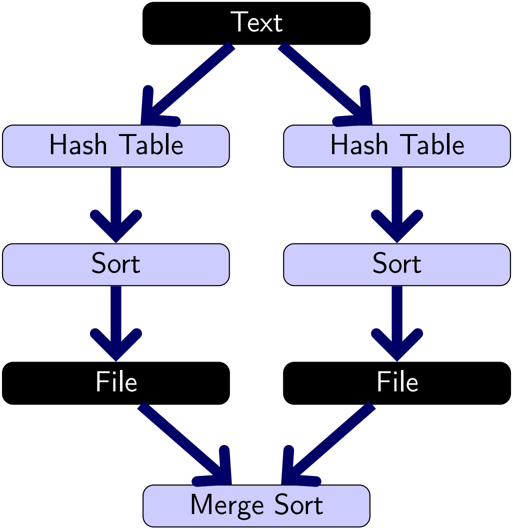
Spill to Disk When RAM Runs Out



Split Data



Split and Merge



Training Problem:
Batch process large number of records.

Solution: Split/merge
Stupid backoff in one pass
Kneser-Ney in three passes

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Batch process large number of records.

Solution: Split/merge
Stupid backoff in one pass
Kneser-Ney in three passes

Training is designed for mutable batch access.
What about queries?

Query

$$\text{stupid}(w_n \mid w_1^{n-1}) = \begin{cases} \dfrac{\text{count}(w_1^n)}{\text{count}(w_1^{n-1})} & \text{if count}(w_1^n) > 0 \\ 0.4\text{stupid}(w_n \mid w_2^{n-1}) & \text{if count}(w_1^n) = 0 \end{cases}$$

stupid(few | is one of the)

count(is one of the few) $= 5$ ✓

count(is one of the) $= 12$

Query

$$\text{stupid}(w_n \mid w_1^{n-1}) = \begin{cases} \dfrac{\text{count}(w_1^n)}{\text{count}(w_1^{n-1})} & \text{if count}(w_1^n) > 0 \\ 0.4\,\text{stupid}(w_n \mid w_2^{n-1}) & \text{if count}(w_1^n) = 0 \end{cases}$$

stupid(periwinkle | is one of the)

$$\begin{aligned} \text{count(is one of the periwinkle)} &= 0 \text{ ✗} \\ \text{count(one of the periwinkle)} &= 0 \text{ ✗} \\ \text{count(of the periwinkle)} &= 0 \text{ ✗} \\ \text{count(the periwinkle)} &= 3 \text{ ✓} \\ \\ \text{count(the)} &= 1000 \end{aligned}$$

Save Memory: Forget Keys

Giant hash table with *n*-grams as keys and counts as values.

Replace the *n*-grams with 64-bit hashes:
Store hash(is one of) instead of "is one of".
Ignore collisions.

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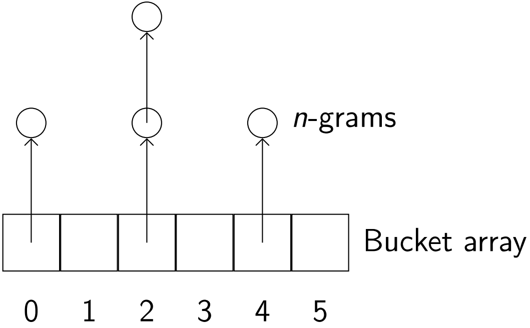
Replace the *n*-grams with 64-bit hashes:
Store hash(is one of) instead of "is one of".
Ignore collisions.

Birthday attack: $\sqrt{2^{64}} = 2^{32}$.
$\implies$ Low chance of collision until $\approx$ 4 billion entries.

Default Hash Table

`boost::unordered_map` and `__gnu_cxx::hash_map`



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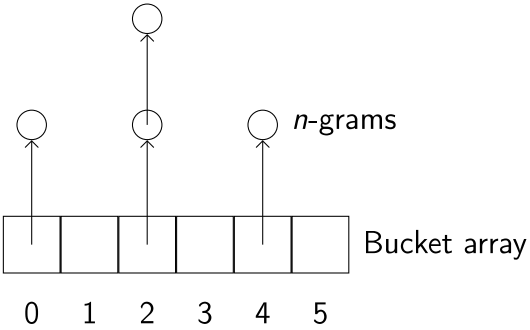

Lookup requires two random memory accesses.

Linear Probing Hash Table

- 1.5 buckets/entry (so buckets = 6).
- Ideal bucket = hash mod buckets.
- Resolve *bucket* collisions using the next free bucket.

Bigrams

Words	Ideal	Hash	Count
iran is	0	`0x959e48455f4a2e90`	3
		`0x0`	0
is one	2	`0x186a7caef34acf16`	5
one of	2	`0xac66610314db8dac`	2
<s> iran	4	`0xf0ae9c2442c6920e`	1
		`0x0`	0

Minimal Perfect Hash Table

Maps every *n*-gram to a unique integer $[0, |n - grams|)$
$\rightarrow$ Use these as array offsets.

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Entries not in the model get assigned offsets
$\rightarrow$ Store a fingerprint of each *n*-gram

Minimal Perfect Hash Table

Maps every n-gram to a unique integer $[0, |n - grams|)$
$\rightarrow$ Use these as array offsets.

Low memory, but potential for false positives

Less Memory: Sorted Array

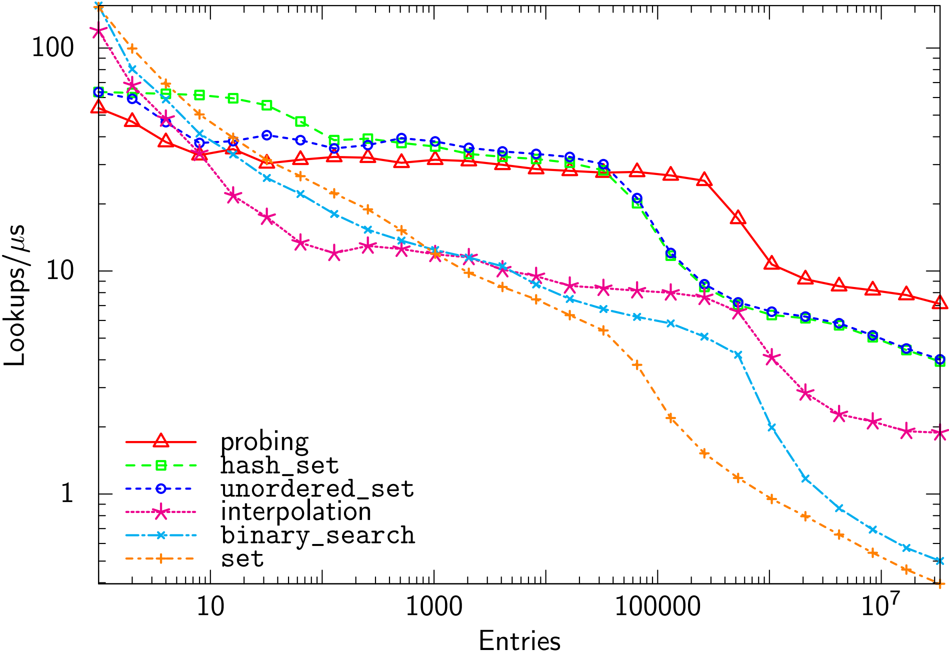
Look up "zebra" in a dictionary.

Binary search

Open in the middle. $O(\log n)$ time.

Interpolation search

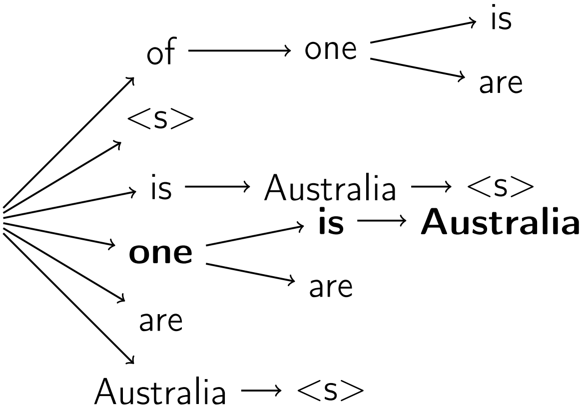
Open near the end. $O(\log \log n)$ time.



100
10
1
Lookups/μs
10
1000
100000
10^7
Entries
probing
hash_set
unordered_set
interpolation
binary_search
set

Trie

Reverse *n*-grams, arrange in a trie.



Saving More RAM

- Quantization: store approximate values
- Collapse probability and backoff

Implementation Summary

Implementation involves sparse mapping

- Hash table
- Probing hash table
- Minimal perfect hash table
- Sorted array with binary or interpolation search

Problem with Backoff:
"in the Ugrasena kingdom"

"Ugrasena" is unseen $\rightarrow$ use unigram to predict "kingdom"

$p(\text{kingdom} \mid \text{in the Ugrasena}) = p(\text{kingdom})$

One rare word of context breaks conditioning.

More Conditioning

Word class/cluster model

Skip-gram: skip one or more context words

Neural networks

More Conditioning

Word class/cluster model

Skip-gram: skip one or more context words

Neural networks

Neural N-grams (aka Feed-Forward)

Language modeling as classification:

Input in the Ugrasena

Output kingdom

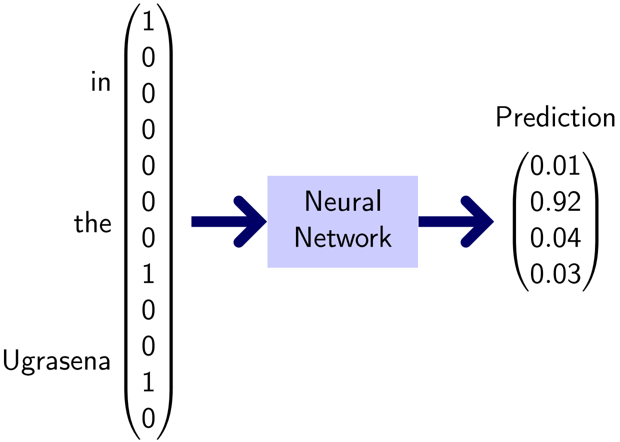
Predict (classify) the next word given preceding words.

Turning Words into Vectors

$$\begin{array}{cccc} \text{in} & \text{kingdom} & \text{Ugrasena} & \text{the} \\ \begin{pmatrix}1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \end{array}$$

Assign each word a unique row

Query: Concatenate Vectors



Issue: Large Vocabulary

The input has $|\text{vocab}| \cdot |\text{history}|$ dimensions.

$$\Longrightarrow$$

The input matrix has $|\text{vocab}| \cdot |\text{history}| \cdot |\text{hidden}|$ parameters.
Say $|\text{vocab}| \approx 1$ billion. About 1 trillion parameters.

Dealing with Vocabulary Size

- **Word vectors**
- Map unpopular words to unknown or part of speech
- Don't use words: characters or short snippets

Turning Words into Vectors

$$\underset{\text{in}}{\begin{pmatrix} -3 \\ 1.5 \\ 6.2 \end{pmatrix}} \quad \underset{\text{kingdom}}{\begin{pmatrix} 2.2 \\ 7.5 \\ -.8 \end{pmatrix}} \quad \underset{\text{Ugrasena}}{\begin{pmatrix} -.1 \\ 0.8 \\ 9.1 \end{pmatrix}} \quad \underset{\text{the}}{\begin{pmatrix} 1.1 \\ 7.0 \\ -.2 \end{pmatrix}}$$

Vectors from the input matrix
. . . or your favorite ACL paper.

Use Word Vectors

Steal word representation from another task (language modeling, IR, SVD).
Reduce from $|\text{vocab}|$ one-hot to ≈ 100 dimensional dense input.

Pro Other task might have more data

Con Does not jointly learn word representation

Output vocabulary

Output is still |vocab|-way classification.

$\Longrightarrow$ Large output matrix, lots to normalize over.

$\Longrightarrow$ Slow evaluation, overfitting.

Dealing with Vocabulary Size

- Word vectors
- Map unpopular words to unknown or part of speech
- **Don't use words: characters or short snippets**

Translation Modeling

$p(\text{bedroom} \mid \text{of the American},$
$\text{de la } \textbf{Chambre} \text{ des représentants des États-Unis})$

[Devin et al 2014]

Conclusion

Language models measure fluency.

Neural networks and backoff are the dominant formalisms.

. . . But you should probably use recurrent networks.